

A dark energy model resulting from a Ricci symmetry revisited

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Abstract

Observations of supernovae of type Ia require dark energy (some unknown exotic ‘*matter*’ of negative pressure) to explain their unexpected faintness. Whereas the simplest and most favoured candidate of dark energy, the Einsteinian cosmological constant, is about 120 orders of magnitude smaller than the theoretically predicted value. Motivated by this problem, a number of models of dynamically decaying dark energy have been proposed by considering different phenomenological laws or potentials of the scalar field, which are more or less ad-hoc. However, it is more advisable to consider the symmetry properties of spacetime rather than the ad-hoc assumptions.

In this view, we consider a model of Robertson-Walker cosmology emerging from a Ricci symmetry which provides consistently an evolving dark energy. We test the model for the recent supernovae Ia data, as well as, the ultracompact radio sources data compiled by Jackson and Dodgson. The model fits the data very well.

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I. INTRODUCTION

It is generally believed that the present expansion of the universe is accelerating due to the presence of some unknown cosmic ‘matter’ with negative pressure generally termed as *dark energy*. The simplest and the most favoured candidate of dark energy is Einstein’s cosmological constant Λ , which is though plagued with horrible fine tuning problems. This happens due to the presence of two values of Λ differing from each other by some 120 orders of magnitudes: the magnitude of Λ at the beginning of inflation and the value given by the present-day observations. This has led a number of cosmologists to consider models of evolving dark energy, by either proposing different phenomenological laws or by considering different potentials of the scalar fields which are more or less ad-hoc. The precise mechanism of the evolution of dark energy, which could be required by some symmetry principle, is not yet known. Indeed, since the basic motivation in these models is to understand the present-day smallness of Λ , they do not provide any natural relation between the magnitude of Λ at the beginning of inflation and the present-day observational value.

It would be worth while to investigate some symmetry principles behind the problem crying out for the evolution of dark energy and thus develop a more realistic and fundamental model for dark energy. Moreover it is always reasonable to consider symmetry properties of spacetime rather than considering ad-hoc assumptions. In this view, we have discovered [1, 2, 3] a model resulting from a contracted Ricci-collineation which, apart from having interesting conservation properties, does provide a dynamical law for decaying Λ . The physical properties of the model have been discussed in [1, 2, 3] and it was found that the model had a credible magnitude-redshift (m - z) relation for the observations of supernovae (SNe) of type Ia from Perlmutter et al. [4]. Since then many observations of SNe Ia have been made, quite many at higher redshifts, by the Hubble Space Telescope. It would be worthwhile to examine how well (or badly) the model fits the new data.

Like the luminosity of a standard candle, the angular size Θ of a standard measuring rod changes with its redshift z in a manner that depends upon the parameters of the model. Hence the Θ - z relation is also proposed as a potential test for cosmological models by Hoyle [5]. Therefore, it would also be worthwhile to examine the Θ - z relation in this model for the dataset of Jackson and Dodgson [6], which is a trustworthy compilation of ultracompact radio sources and has been already used to test different cosmological models [6, 7]. In

the following section we describe the model in brief for ready reference and to derive the observational relations. More details can be found in [3].

II. THE MODEL

As we are going to consider a dynamical Λ in Einstein's theory, it would be worthwhile to mention a general result which holds irrespective of the dynamics of Λ : the empty spacetime of de Sitter cannot be a solution of general relativity with a dynamical $\Lambda(t)$ [8]. This follows from the divergence of the field equation $[R^{ij} - \frac{1}{2}Rg^{ij}]_{;j} = 0 = [T_{(m)}^{ij} - g^{ij}\Lambda(t)/8\pi G]_{;j}$. Obviously a solution with a dynamical $\Lambda(t)$ is possible only if $T_{(m)}^{ij} \neq 0$ (and $T_{(m);j}^{ij} \neq 0$). We assume that the universe is homogeneous and isotropic, represented by the Robertson-Walker (RW) metric, and its dynamics is given by the Einstein field equations

$$-\frac{\ddot{S}}{S} = \frac{4\pi G}{3}(\rho_{\text{tot}} + 3p_{\text{tot}}), \quad (1)$$

$$\frac{\dot{S}^2}{S^2} + \frac{k}{S^2} = \frac{8\pi G}{3}\rho_{\text{tot}}, \quad (2)$$

where S and k are respectively the scale factor and curvature index appearing in the RW metric; and $\rho_{\text{tot}} = \rho_m + \rho_\Lambda \equiv \rho_m + \Lambda(t)/8\pi G$, $p_{\text{tot}} = p_m + p_\Lambda = p_m - \rho_\Lambda$, with Λ representing the cosmological term.

It is well known that collineations of Ricci tensor (R_{ij}) have interesting symmetry properties and lead to useful conservation laws in general relativity [1, 2, 3, 9]. The Ricci collineation along a vector η_i is defined by vanishing Lie-derivative of R_{ij} along η_i : $\mathcal{L}_\eta R_{ij} = 0$. It has been shown [1, 2] that in general relativity the contracted Ricci-collineation along the fluid flow u^i (normalized velocity 4-vector), i.e., $g^{ij}\mathcal{L}_u R_{ij} = 0$, leads to the conservation of generalized *momentum density*:

$$\{(\rho_{\text{tot}} + 3p_{\text{tot}})u^j\}_{;j} = 0. \quad (3)$$

For the RW metric, the conservation law (3) leads to the conservation of the total active gravitational mass of a comoving sphere of radius S :

$$(\rho_{\text{tot}} + 3p_{\text{tot}})S^3 = \text{constant} = A \quad (\text{say}) \quad (4)$$

(A detailed discussion elaborating on the meaning of (4) has been done in [3]. Consequences of the resulting models for the case $\Lambda = 0$ have been discussed in [1].) As there are 4

unknowns ρ_m , p_m , ρ_Λ ($= \Lambda(t)/8\pi G$) and the scale factor S , the equations (1), (2) and (4), together with the usual barotropic equation of state $p_m/\rho_m = \text{constant} = w$ ($0 \leq w \leq 1$) for the matter source, provide a unique solution of the model. By using (4) in the Raychaudhuri equation (1) and integrating the resulting equation, we get

$$\frac{\dot{S}^2}{S^2} = \frac{8\pi G A}{3S^3} + \frac{B}{S^2}, \quad (5)$$

supplying the dynamics of the scale factor, where B is a constant of integration. Equations (2), (4) and (5) supply the unique dynamics of dark energy density ρ_Λ and ρ_m as

$$\rho_\Lambda = \frac{1}{(1+w)} \left[\frac{wA}{S^3} + \frac{(1+3w)(B+k)}{8\pi G S^2} \right], \quad (6)$$

$$\rho_m = \frac{1}{(1+w)} \left[\frac{A}{S^3} + \frac{(B+k)}{4\pi G S^2} \right]. \quad (7)$$

It must be noted that we have not assumed the conservation of the matter source which is usually done through the additional assumption of no interaction (minimal coupling) between different source fields (except for the case with a constant Λ which is consistent with the idea of minimal coupling), which though seems ad-hoc and nothing more than a simplifying assumption. On the contrary, interaction is more natural and is a fundamental principle. Let us recall that the only constraint on the source terms, which is imposed by Einstein's equation (through the Bianchi identities), is the conservation of the sum of all the energy-momentum tensors, individually they are not conserved: $[R^{ij} - \frac{1}{2}Rg^{ij}]_{;j} = 0 = [T_{(m)}^{ij} + T_{(\Lambda)}^{ij} + T_{(\phi)}^{ij} + \dots]_{;j}$, implying creation or annihilation for the case $\Lambda(t)$.

It may be noted that the evolution of ρ_Λ , as given by equation (6), is a function of the equation of state of matter. This may be regarded as a kind of generalization of the ansatz $\Lambda \propto S^{-2}$ proposed by many authors [10], which is obtained in the present model in the present phase of evolution of the universe ($w = 0$).

In order to study the Θ - z and the m - z relations in the model, we can rewrite equations (5-7) by specifying the constants A and B in terms of the cosmological parameters in the present phase of evolution: $A = (\Omega_0 - 2\Omega_{\Lambda 0})3S_0^3 H_0^2/8\pi G$, $B = (2\Omega_{\Lambda 0} - \Omega_0 + 1)H_0^2 S_0^2$, giving

$$\Omega_\Lambda(z) = \Omega_{\Lambda 0}(1+z)^2 \frac{H_0^2}{H^2(z)}, \quad (8)$$

$$\Omega_m(z) = [(\Omega_{m0} - 2\Omega_{\Lambda 0})(1+z)^3 + 2\Omega_{\Lambda 0}(1+z)^2] \frac{H_0^2}{H^2(z)}, \quad (9)$$

where

$$H(z) = H_0[(\Omega_{m0} - 2\Omega_{\Lambda0})(1+z)^3 - (\Omega_{m0} - 2\Omega_{\Lambda0} - 1)(1+z)^2]^{1/2}. \quad (10)$$

Here Ω_i are, as usual, the energy density of different source components of the cosmological fluid in units of the critical density $3H^2/8\pi G$ (i denoting matter (m), cosmological term (Λ), etc.). The subscript ‘0’ denotes the value of the quantity at the present epoch.

The angular size-redshift (Θ - z) relation in the model is given by Hoyle’s formula

$$\Theta = \frac{d(1+z)}{rS_0} \text{ radian}, \quad (11)$$

which relates the apparent angular diameter Θ of the source (of redshift z located at a coordinate distance r) with its absolute angular size d (presumably same for all sources). The coordinate distance r can be calculated from the RW metric according to its curvature parameter k :

$$r = \begin{cases} \sin\left(\frac{1}{S_0} \int_0^z \frac{dz'}{H(z')}\right), & \text{when } k = 1 \\ \frac{1}{S_0} \int_0^z \frac{dz'}{H(z')}, & \text{when } k = 0 \\ \sinh\left(\frac{1}{S_0} \int_0^z \frac{dz'}{H(z')}\right), & \text{when } k = -1. \end{cases} \quad (12)$$

The present value of the scale factor S_0 , appearing in equations (11, 12), which measures the present curvature of spacetime, can be calculated from

$$S_0 = H_0^{-1} \sqrt{\frac{k}{(\Omega_{m0} + \Omega_{\Lambda0} - 1)}}. \quad (13)$$

As the measured angular sizes of the radio sources in the dataset from Jackson and Dodgson are given in units of milli arc second (mas), we rewrite equation (11) as

$$\Theta(z; \Omega_i, dh_0) = 0.0688 \times dh_0 \sqrt{\frac{(\Omega_{m0} + \Omega_{\Lambda0} - 1)}{k}} \frac{(1+z)}{r} \text{ mas}, \quad (14)$$

where d is measured in pc (par sec) and h_0 is the present value of the Hubble constant H_0 measured in units of $100 \text{ Km s}^{-1} \text{ Mpc}^{-1}$. We are now able to calculate the angular size $\Theta(z, \Omega_i, dh_0)$ of a radio source at a given redshift z predicted by the model (for a given set of the parameters Ω_i, dh_0) by using equations (10, 12-14).

We also recall that the usual magnitude-redshift (m - z) relation in a homogeneous and isotropic model (based on the RW metric) is given by

$$m(z; \Omega_i, \mathcal{M}) = \mathcal{M} + 5 \log\{H_0 d_L(z; \Omega_i)\}, \quad (15)$$

where m is the apparent magnitude of a SN of redshift z located at the coordinate distance r , $\mathcal{M} \equiv M - 5 \log H_0 + 25$, M is the absolute magnitude (presumably same for all SNe Ia), and d_L is the luminosity distance of the SN given by

$$d_L(z; \Omega_i) = (1 + z)S_0 r \quad \text{Mpc.} \quad (16)$$

Now we can calculate the magnitude $m(z, \Omega_i, \mathcal{M})$ of a SN at a given redshift z predicted by the model (for a given set of the parameters Ω_i, \mathcal{M}) by using equations (10, 12, 13, 15, 16).

In order to fit the model to the observations, we calculate χ^2 according to

$$\chi^2 = \sum_{i=1}^N \left[\frac{X_i^{\text{obs}} - X^{\text{pred}}(z_i)}{\sigma_{X_i^{\text{obs}}}} \right]^2, \quad (17)$$

where X_i^{obs} is the observed value of the observable, $X^{\text{pred}}(z_i)$ is its predicted value at the redshift z_i , $\sigma_{X_i^{\text{obs}}}$ is the uncertainty in the observed value X_i^{obs} and N is the number of data points (or bins). (X stands for Θ and m respectively for the data of radio sources and SNe Ia.)

III. FITTING THE MODEL TO THE RADIO SOURCES DATA

We use the sample of 256 ultracompact radio sources compiled by Jackson and Dodgson [6]. This sample of 256 radio sources with z in the range 0.5 to 3.8 was selected by them from a bigger sample of 337 ultracompact radio sources originally compiled by Gurvits [11]. These sources, of angular sizes of the order of a few milliarcseconds (ultracompact), were measured by the *very long-baseline interferometry*. The points of the sample of Jackson and Dodgson are short-lived quasars deeply embedded inside the galactic nuclei, which are expected to be free from evolution on a cosmological time scale and thus comprise a set of standard rods, at least in a statistical sense. Jackson and Dodgson binned their sample into 16 redshift bins, each bin containing 16 sources. We fit the present model to this sample by calculating χ^2 according to (17) and minimize it with respect to the free parameters Ω_{m0} , $\Omega_{\Lambda0}$ and dh_0 . The global minimum is obtained for the values (with the constraint $\Omega_{m0} \geq 0$)

$$\Omega_{m0} = 0.68, \Omega_{\Lambda0} = 0.45, dh_0 = 6.50 \text{ with } \chi^2 = 13.00$$

at 13 degrees of freedom (DoF). This represents a very good fit, with the goodness-of-fit probability $Q = 44.8\%$ (see the Appendix for an explanation of Q). We further note that the

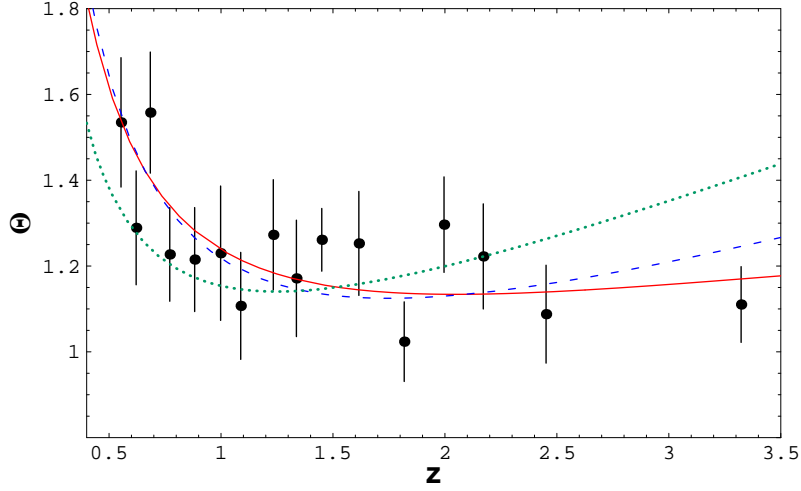


FIG. 1: Some best-fitting models to the ultracompact radio sources data from Jackson & Dodgson are shown. The solid curve corresponds to the best-fitting dark energy model $\rho_\Lambda \propto S^{-2}$ with $\Omega_{m0} = 1 - \Omega_{\Lambda0} = 0.62$, the dashed curve corresponds to the best-fitting standard Λ CDM model with $\Omega_{m0} = 1 - \Omega_{\Lambda0} = 0.21$ and the dotted curve corresponds to the best-fitting Einstein-de Sitter model ($\Omega_{m0} = 1 - \Omega_{\Lambda0} = 1$).

solution for the minimum χ^2 is very degenerate in the parameter space and the parameters wander near the global minimum of χ^2 in almost a flat valley of some complicated topology. For example, the best-fitting flat model ($\Omega_{\Lambda0} = 1 - \Omega_{m0}$) is obtained as

$$\Omega_{m0} = 0.62 \pm 0.03, dh_0 = 6.46 \pm 0.31, \chi^2/\text{DoF} = 13.09/14, Q = 51.9\%,$$

which represents a slightly better fit (as the number of DoF is increased). It may be mentioned that the value $\Omega_{m0} = 0.62$ estimated from these data is in good agreement with the results estimated (in the following section) from the recent observations of SNe of type Ia. It should be noted that the best-fitting solution $\Omega_{m0} = 0.62$ gives a mildly accelerating expansion of the universe at the present epoch: the deceleration parameter $q_0 \equiv \Omega_{m0}/2 - \Omega_{\Lambda0} = -0.07$. In order to compare, we find that the best-fitting concordance model (flat standard Λ CDM model with a constant Λ) to this dataset is obtained as

$$\Omega_{m0} = 0.21 \pm 0.08, dh_0 = 7.25 \pm 0.55, \chi^2/\text{DoF} = 16.03/14, Q = 31.1\%,$$

which also represents a good fit. One may note that the value $\Omega_{m0} = 0.62$ obtained for the model $\rho_{\Lambda} \propto S^{-2}$ is higher than $\Omega_{m0} = 0.21$ obtained for the standard Λ CDM model. However, one should note that the other precise observations, which one would expect to be consistent with any model, are the measurements of the temperature anisotropy of CMB made by the WMAP experiments [12], whose only apparent prediction is $\Omega_{m0} + \Omega_{\Lambda0} = 1$ [13]. For this reason, and also motivated by theoretical considerations required by inflation and flatness problem, we assume spatial flatness henceforth. For curiosity, we test the Einstein-de Sitter model ($\Omega_{m0} = 1, \Lambda = 0$) against the radio sources data: the best fitting solution is obtained as

$$dh_0 = 4.91 \pm 0.11, \chi^2/\text{DoF} = 28.82/15, Q = 1.69\%,$$

which can be rejected by the data only at 98.3% confidence level. In Figure 1, we have shown some models obtained from our fitting procedure and compared them with the data.

IV. FITTING THE MODEL TO THE RECENT SNE IA DATA

It was already shown that the model explained the data of SNe of type Ia from Perlmutter et al. successfully [4]. Since then many SNe of type Ia at higher redshifts have been discovered. Extending our earlier work further, we now examine how the model fits the updated *gold sample* of Riess et al. [14]. In addition to having previously discovered SNe Ia, this sample (of 182 SNe Ia in total) contains 23 SNe Ia at $z \geq 1$ recently discovered by the Hubble Space Telescope and is claimed to have a high-confidence-quality of spectroscopic and photometric record for the individual SNe.

We find that the present model has an excellent fit to this data, comparable with the concordance model. The best-fitting concordance model is obtained as

$$\Omega_{m0} = 0.34 \pm 0.04, \mathcal{M} = 43.40 \pm 0.03, \chi^2/\text{DoF} = 158.75/180, Q = 87.1\%,$$

an excellent fit indeed! The model $\Lambda \propto S^{-2}$ provides a similar fit:

$$\Omega_{m0} = 0.58 \pm 0.02, \mathcal{M} = 43.46 \pm 0.03, \chi^2/\text{DoF} = 167.90/180, Q = 73.2\%.$$

The Einstein-de Sitter model does not fit the data well:

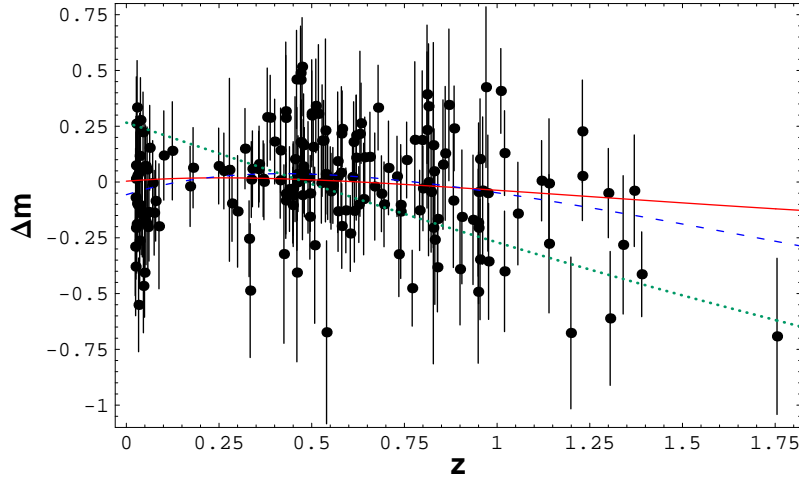


FIG. 2: Modified Hubble diagram of new ‘gold sample’ of SNe Ia [14] minus a fiducial model ($\Omega_{m0} = \Omega_{\Lambda0} = 0$): The relative magnitude ($\Delta m \equiv m - m_{\text{fiducial}}$) is plotted for some best-fitting models. The solid curve corresponds to the dark energy model $\rho_{\Lambda} \propto S^{-2}$ with $\Omega_{m0} = 1 - \Omega_{\Lambda0} = 0.58$, the dashed curve corresponds to the standard concordance model with $\Omega_{m0} = 1 - \Omega_{\Lambda0} = 0.34$ and the dotted curve corresponds to the Einstein-de Sitter model.

$$\mathcal{M} = 43.72 \pm 0.02, \chi^2/\text{DoF} = 283.40/181, Q = 1.7 \times 10^{-4}\%.$$

These models have been shown in Figure 2. In order to have a visual comparison of the fits of different models to the actual data points, we magnify their differences by plotting the relative magnitude with respect to a fiducial model $\Omega_{m0} = \Omega_{\Lambda0} = 0$ (which also has a good fit: $\chi^2/\text{DoF} = 174.29/181, Q = 62.6\%$).

V. CONCLUSIONS

In order to test the consistency of the cosmological models with observations as well as to estimate the different cosmological parameters, data on SNe of type Ia and radio sources have been used by several authors. We use the recent gold sample of SNe Ia from Riess et al. and the sample of 256 ultracompact radio sources (of angular sizes of the order of a few milliarcseconds) compiled by Jackson and Dodgson to test a model of dark energy which

results consistently from the contracted Ricci-collineation along the fluid flow vector. We find that the model has excellent fits to both the data sets and the estimated parameters are also in good agreement.

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APPENDIX

Though there is not a clearly defined value of χ^2/DoF for an acceptable fit, however it is obvious from equation (17) that if the model represents the data correctly, the difference between the predicted angular size/magnitude and the observed one at each data point should be roughly the same size as the measurement uncertainties and each data point will contribute to χ^2 roughly one, giving the sum roughly equal to the number of data points N (more correctly $N - \text{number of fitted parameters} \equiv \text{number of degrees of freedom 'DoF'}$). This is regarded as a ‘*rule of thumb*’ for a *moderately* good fit. If χ^2 is large, the fit is bad. However we must quantify our judgment and decision about the *goodness-of-fit*, in the absence of which, the estimated parameters of the model (and their estimated uncertainties) have no meaning at all. An independent assessment of the goodness-of-fit of the data to the model is given in terms of the χ^2 -*probability*: if the fitted model provides a typical value of χ^2 as x at n DoF, this probability is given by

$$Q(x, n) = \frac{1}{\Gamma(n/2)} \int_{x/2}^{\infty} e^{-u} u^{n/2-1} du. \quad (\text{A.1})$$

Roughly speaking, it measures *the probability that the model does describe the data genuinely and any discrepancies are mere fluctuations which could have arisen by chance*. To be more precise, $Q(x, n)$ gives the probability that a model which does fit the data at n DoF, would give a value of χ^2 as large or larger than x . If Q is very small, the apparent discrepancies are unlikely to be chance fluctuations and the model is ruled out. For example, if we get a $\chi^2 = 20$ at 5 DoF for some model, then the hypothesis that *the model describes the data genuinely* is unlikely, as the probability $Q(20, 5) = 0.0012$ is very small. It may however be noted that the χ^2 -probability strictly holds only when the models are linear in their parameters and the measurement errors are normally distributed. It is though common, and usually not too wrong, to assume that the χ^2 -distribution holds even for models which

are not strictly linear in their parameters, and for this reason, the models with a probability as low as $Q > 0.001$ are usually deemed acceptable [15].

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